

Supplemental WS #8

$$\begin{aligned}
 \textcircled{1} \quad \sin 75^\circ &= \sin (45^\circ + 30^\circ) \\
 &= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \sin 30^\circ \\
 &= \left(\frac{\sqrt{2}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right) \left(\frac{1}{2}\right) \\
 &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\
 &= \frac{\sqrt{6} + \sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad \cos 15^\circ &= \cos (45^\circ - 30^\circ) \\
 &= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\
 &= \left(\frac{\sqrt{2}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right) \left(\frac{1}{2}\right) \\
 &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\
 &= \frac{\sqrt{6} + \sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad \sin \frac{\pi}{12} &= \sin \left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) \\
 &= \sin \left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\
 &= \sin \frac{\pi}{3} \cdot \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \cdot \sin \frac{\pi}{4} \\
 &= \left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right) \left(\frac{\sqrt{2}}{2}\right) \\
 &= \frac{\sqrt{6} - \sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \quad \tan 105^\circ &= \tan (60^\circ + 45^\circ) \\
 &= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \cdot \tan 45^\circ} \\
 &= \frac{(\sqrt{3}) + (1)}{1 - (\sqrt{3})(1)} \\
 &= \frac{\sqrt{3} + 1}{1 - \sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \quad \cos \frac{7\pi}{12} &= \cos \left(\frac{4\pi}{12} + \frac{3\pi}{12} \right) \\
 &= \cos \left(\frac{\pi}{3} + \frac{\pi}{4} \right) \\
 &= \cos \frac{\pi}{3} \cos \frac{\pi}{4} - \sin \frac{\pi}{3} \sin \frac{\pi}{4} \\
 &= \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right) - \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) \\
 &= \frac{\sqrt{2} - \sqrt{6}}{4}
 \end{aligned}$$

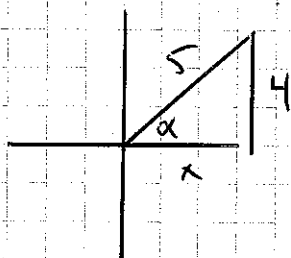
$$\begin{aligned}
 \textcircled{6} \quad \sin 105^\circ &= \sin (60^\circ + 45^\circ) \\
 &= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\
 &= \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right) + \left(\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right) \\
 &= \frac{\sqrt{6} + \sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{7} \quad \cos \frac{5\pi}{12} &= \cos \left(\frac{3\pi}{12} + \frac{2\pi}{12} \right) \\
 &= \cos \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \\
 &= \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \sin \frac{\pi}{6} \\
 &= \left(\frac{\sqrt{2}}{2} \right) \left(\frac{\sqrt{3}}{2} \right) - \left(\frac{\sqrt{2}}{2} \right) \left(\frac{1}{2} \right) \\
 &= \frac{\sqrt{6} - \sqrt{2}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{8} \quad \tan \frac{\pi}{12} &= \tan \left(\frac{3\pi}{12} - \frac{2\pi}{12} \right) \\
 &= \tan \left(\frac{\pi}{4} - \frac{\pi}{6} \right) \\
 &= \frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{4} \cdot \tan \frac{\pi}{6}} \\
 &= \frac{(1) - \left(\frac{\sqrt{3}}{3} \right)}{1 + (1) \left(\frac{\sqrt{3}}{3} \right)} \\
 &= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} \\
 &= \frac{3 - \sqrt{3}}{3 + \sqrt{3}}
 \end{aligned}$$

9

$$\sin \alpha = \frac{4}{5}$$



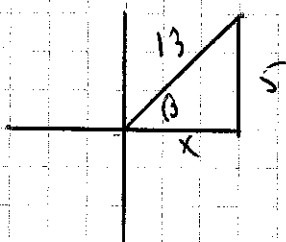
$$x^2 + 4^2 = 5^2$$

$$x = 3$$

∴

$$\cos \alpha = \frac{3}{5}$$

$$\sin \beta = \frac{5}{13}$$



$$x^2 + 5^2 = 13^2$$

$$x = 12$$

∴

$$\cos \beta = \frac{12}{13}$$

$$\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

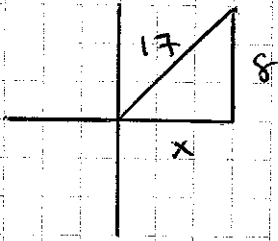
$$= \left(\frac{4}{5}\right) \left(\frac{12}{13}\right) + \left(\frac{3}{5}\right) \left(\frac{5}{13}\right)$$

$$= \frac{48}{65} + \frac{15}{65}$$

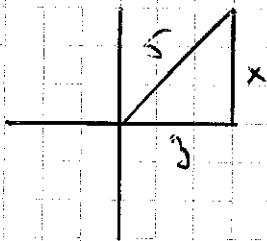
$$= \frac{63}{65}$$

(3)

$$\sin \alpha = \frac{8}{17}$$



$$\cos \beta = \frac{3}{5}$$



$$x^2 + 8^2 = 17^2$$

$$x = \sqrt{\quad}$$

$$\therefore \cos \alpha = \frac{9}{17}$$

$$3^2 + x^2 = 5^2$$

$$x = 4$$

$$\therefore \sin \beta = \frac{4}{5}$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

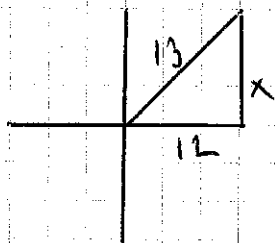
$$= \left(\frac{9}{17}\right) \left(\frac{3}{5}\right) + \left(\frac{8}{17}\right) \left(\frac{4}{5}\right)$$

$$= \frac{27}{85} + \frac{32}{85}$$

$$= \frac{59}{85}$$

⑪

$$\cos \alpha = \frac{12}{13}$$

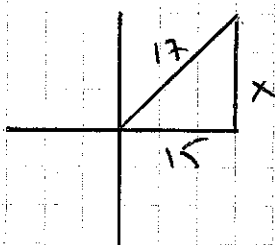


$$x^2 + 12^2 = 13^2$$

$$x = 5$$

$$\therefore \sin \alpha = \frac{5}{13}$$

$$\cos \beta = \frac{15}{17}$$



$$x^2 + 15^2 = 17^2$$

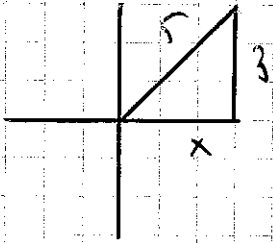
$$x = 8$$

$$\therefore \sin \beta = \frac{8}{17}$$

$$\begin{aligned} \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= \left(\frac{5}{13}\right) \left(\frac{15}{17}\right) - \left(\frac{12}{13}\right) \left(\frac{8}{17}\right) \\ &= \frac{75}{221} - \frac{96}{221} \\ &= -\frac{21}{221} \end{aligned}$$

12

$$\sin \alpha = \frac{3}{5}$$

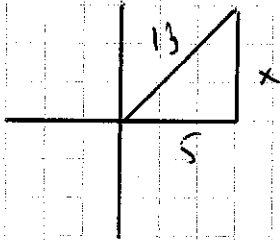


$$x^2 + 3^2 = 5^2$$

$$x = 4$$

$$\therefore \tan \alpha = \frac{3}{4}$$

$$\cos \beta = \frac{5}{13}$$



$$x^2 + 5^2 = 13^2$$

$$x = 12$$

$$\therefore \tan \beta = \frac{12}{5}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$= \frac{\left(\frac{3}{4}\right) + \left(\frac{12}{5}\right)}{1 - \left(\frac{3}{4}\right)\left(\frac{12}{5}\right)}$$

$$= \frac{\frac{15}{20} + \frac{48}{20}}{20 - \frac{36}{20}}$$

$$= \frac{\frac{63}{20}}{\frac{20 \cdot 20 - 36}{20}}$$

$$= \frac{63}{20 \cdot 17}$$

$$= \frac{63}{340}$$

$$= \frac{63}{340}$$